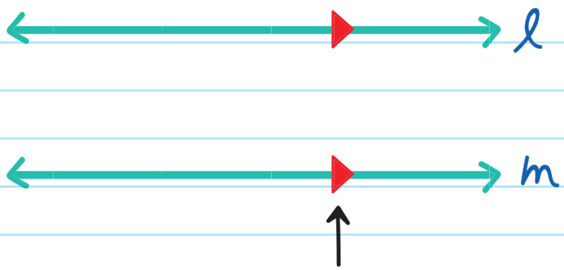
Chapter 7

PARALLEL LINES AND TRANSVERSALS

PARALLEL LINES are lines on the same plane that never meet (intersect). They're indicated with arrows.



arrows on lines show the lines are parallel

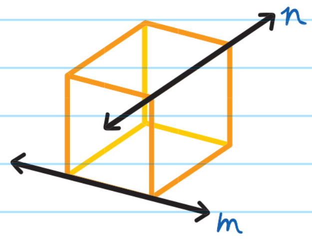
Parallel lines are the same distance from each other over their entire lengths.

This notation || is used to show parallel lines: $\ell \parallel m$

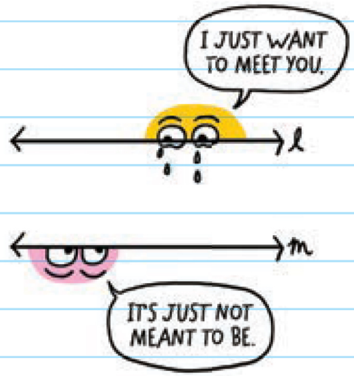
|| is the symbol for "is parallel to"

∦ is the symbol for "is not parallel to"

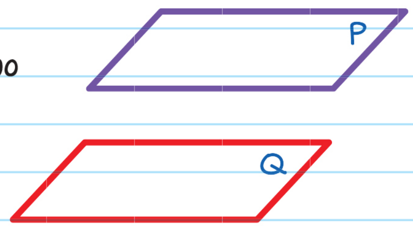
SKEW LINES are two lines, on different planes, that never meet.



n and m are skew lines



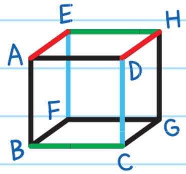
PARALLEL PLANES are two planes that never intersect.



Plane P || Plane Q

Two segments or rays are parallel if the lines that contain them are parallel, and they are skew if the lines that contain them are skew.

EXAMPLES:

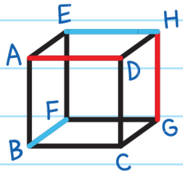


Parallel Segments

$\overline{AE} \parallel \overline{DH}$

$\overline{EF} \parallel \overline{DC}$

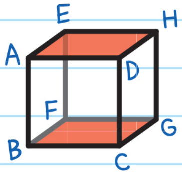
$\overline{BC} \parallel \overline{EH}$



Skew Segments

$\overline{AD}$ and $\overline{HG}$

$\overline{BF}$ and $\overline{EH}$

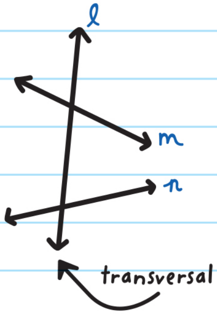


Parallel Planes

Plane AEH $\parallel$ Plane BCG

TRANSVERSALS

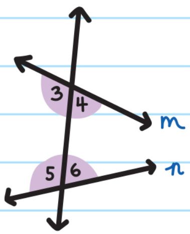
A **TRANSVERSAL** is a line that intersects two or more lines.



The angles that are formed by a transversal and the lines it intersects have special names.

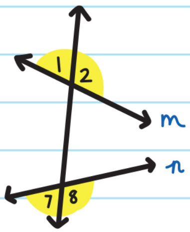
INTERIOR ANGLES are all the angles between the lines intersected by the transversal.

Interior angles: $\angle 3, \angle 4, \angle 5, \angle 6$

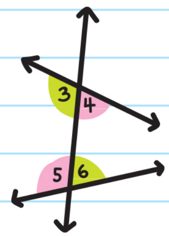
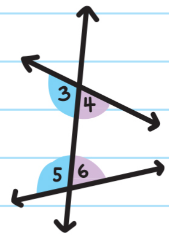
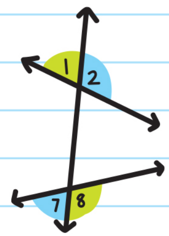


EXTERIOR ANGLES are all the angles that are not between the lines intersected by the transversal.

Exterior angles: $\angle 1, \angle 2, \angle 7, \angle 8$

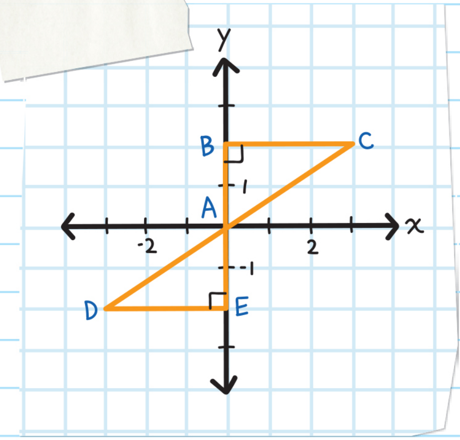


TRANSVERSAL ANGLE PAIRS

ANGLE PAIR	EXAMPLE	CHARACTERISTICS
ALTERNATE INTERIOR ANGLES	 $\angle 3$ and $\angle 6$ $\angle 4$ and $\angle 5$	interior angles on opposite sides of the transversal
SAME-SIDE INTERIOR ANGLES (CORRESPONDING INTERIOR ANGLES)	 $\angle 3$ and $\angle 5$ $\angle 4$ and $\angle 6$	interior angles on the same side of the transversal
ALTERNATE EXTERIOR ANGLES	 $\angle 1$ and $\angle 8$ $\angle 2$ and $\angle 7$	exterior angles on opposite sides of the transversal

EXAMPLE:

Determine if the two triangles are congruent.



Corresponding sides:

Since $AB = 2$ units and $AE = 2$ units

$\overline{AB} \cong \overline{AE}$

Since $BC = 3$ units and $DE = 3$ units,

$\overline{DE} \cong \overline{BC}$

Included angle:

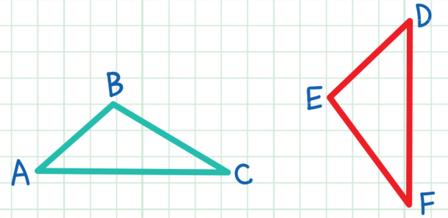
$m\angle DEA = 90°$ and $m\angle CBA = 90°$, so

$\angle DEA \cong \angle CBA$

Therefore, by the SIDE-ANGLE-SIDE CONGRUENCE POSTULATE, $\triangle DEA \cong \triangle CBA$.

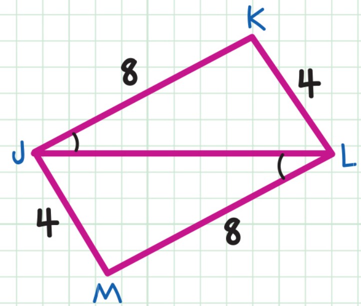
CHECK YOUR KNOWLEDGE

1. Given $\triangle ABC \cong \triangle DEF$, state the congruent corresponding sides and angles.



For questions 2–6, determine if the given triangles are congruent. If so, write a congruence statement and include the postulate (SSS or SAS) it demonstrates.

2. $\triangle JKL$ and $\triangle LMJ$



MORE QUESTIONS

$8 \times 3 = \frac{2}{\cancel{3}} BF \times \cancel{3}$	Multiply both sides by 3.
$24 = 2 \times BF$	Divide both sides by 2.
$BF = 12$	

We can now find GF using the **SEGMENT ADDITION POSTULATE**:

$BF = BG + GF$

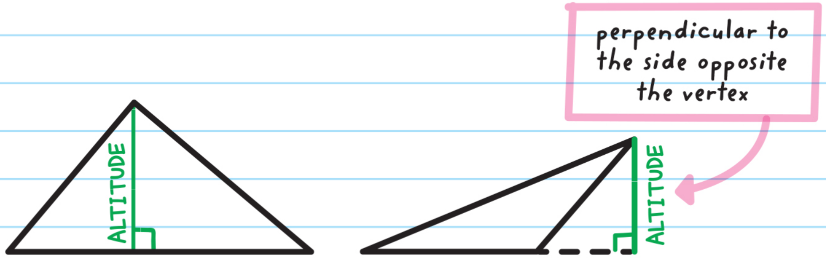
$12 = 8 + GF$

$GF = 4$

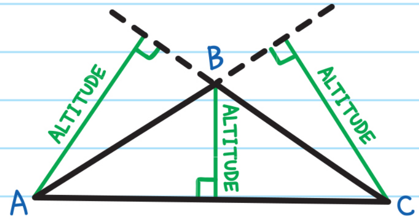
If you wanted to balance a triangle plate on one finger, you would need to place your finger on the centroid to balance it. This point is called the **center of gravity**—the point where the weight is equally balanced.

ALTITUDE AND ORTHOCENTER

The **ALTITUDE** of a triangle is the line segment from a vertex to the opposite side, and perpendicular to that side. An altitude can be outside or inside the triangle.

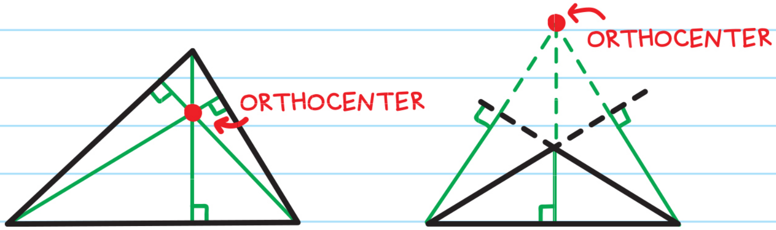


Every triangle has three altitudes.



The point where the altitudes of a triangle meet is the **ORTHOCENTER**.

The orthocenter can be outside or inside the triangle.



Chapter 35

TRIGONOMETRIC RATIOS

TRIGONOMETRY is used to find measures in triangles.

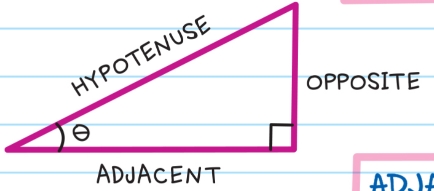
> **TRIGONOMETRY**
> the study of the relationship between side lengths and angles in triangles.

Trigonometry is from the Greek
- *trigonon* = triangle
- *metron* = measure

Important right triangle terms:

HYPOTENUSE the longest side

OPPOSITE the leg that is opposite angle θ



ADJACENT the leg that is next to angle θ

θ (**THETA**) is a Greek letter used to represent an angle.

The trigonometric functions **SINE (SIN)**, **COSINE (COS)**, and **TANGENT (TAN)** are each a ratio of sides of a right triangle. They are used to find unknown angle measures or side lengths of a right triangle.

Sine:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Cosine:

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Tangent:

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Remember the trigonometric functions by using:

SOH-CAH-TOA

Sin = Opposite/Hypotenuse

Cos = Adjacent/Hypotenuse

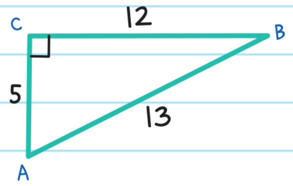
Tan = Opposite/Adjacent

OR

SOH-CAH-TOA

Sam's Old Hairy Cat Ate His Tub Of Applesauce.

EXAMPLE: Find sin A, cos A, tan A, sin B, cos B, and tan B.



$\sin A = \frac{\text{opposite } \angle A}{\text{hypotenuse}} = \frac{12}{13}$

$\cos A = \frac{\text{adjacent to } \angle A}{\text{hypotenuse}} = \frac{5}{13}$

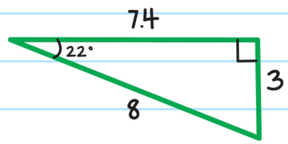
$\tan A = \frac{\text{opposite } \angle A}{\text{adjacent to } \angle A} = \frac{12}{5}$

$\sin B = \frac{\text{opposite } \angle B}{\text{hypotenuse}} = \frac{5}{13}$

$\cos B = \frac{\text{adjacent to } \angle B}{\text{hypotenuse}} = \frac{12}{13}$

$\tan B = \frac{\text{opposite } \angle B}{\text{adjacent to } \angle B} = \frac{5}{12}$

EXAMPLE: Find sin 22°.



$\sin 22° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{8}$

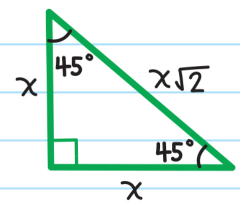
SPECIAL RIGHT TRIANGLES

A special right triangle is a triangle with a feature (angle or side length) measure that makes calculations easier or for which formulas exist. The two most common right triangle measurements are:

45°-45°-90°

hypotenuse = leg $\times \sqrt{2}$

All 45°-45°-90° triangles are similar.

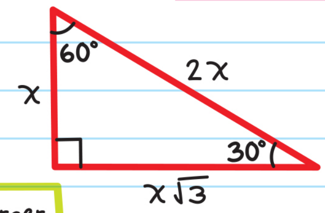


RATIO OF SIDES

$1 : 1 : \sqrt{2}$

45°-45°-90° is an isoceles right triangle.

30°-60°-90°



RATIO OF SIDES

$1 : 2 : \sqrt{3}$

opposite larger angle (60°)

opposite smaller angle (30°)

longer leg = shorter leg $\times \sqrt{3}$

hypotenuse = shorter leg $\times$ 2

All 30°-60°-90° triangles are similar.